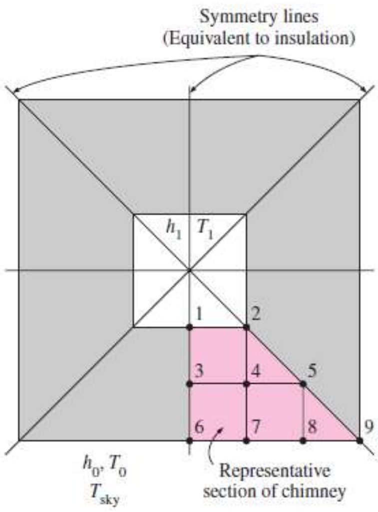
$q_{out} = 3019 + 1214.6 + 600.7 = 4834.3$ W/m.

Example 4.2: Irregular Boundaries

Hot combustion gases of a furnace are flowing through a square chimney made of concrete ($k = 1.4$ W/m · °C). The flow section of the chimney is 20 cm × 20 cm, and the thickness of the wall is 20 cm. The average temperature of the hot gases in the chimney is $T_i = 300$°C, and the average convection heat transfer coefficient inside the chimney is $h_i = 70$ W/m^2 · °C. The chimney is losing heat from its outer surface to the ambient air at $T_o = 20$°C by convection with a heat transfer coefficient of $h_o = 21$ W/m^2 · °C and to the sky by radiation. The emissivity of the outer surface of the wall is $\varepsilon = 0.9$, and the effective sky temperature is estimated to be 260 K. Using the finite difference method with $\Delta x = \Delta y = 10$ cm and taking full advantage of symmetry, determine the temperatures at the nodal points of a cross section and the rate of heat loss for a 1-m-long section of the chimney.



Node 1. On the inner boundary, subjected to convection,

$$-k\frac{\Delta y}{2}\frac{(T_1-T_2)}{\Delta x} - k\frac{\Delta x}{2}\frac{(T_1-T_3)}{\Delta y} = h_i\frac{\Delta x}{2}(T_1 - T_i)$$

Taking $\Delta x = \Delta y$, it simplifies to

$$-k\frac{(T_1-T_2)}{\Delta x} - k\frac{(T_1-T_3)}{\Delta x} = h_i(T_1 - T_i)$$

$$T_1\left(\frac{\Delta x h_i}{k} + 2\right) = \frac{\Delta x h_i}{k}T_i + (T_2 + T_3)$$

$$T_1 = \frac{\left(\frac{\Delta x h_i}{k}\right)T_i + (T_2 + T_3)}{\left(\frac{\Delta x h_i}{k} + 2\right)}$$

Node 2. On the inner boundary, subjected to convection, Figure

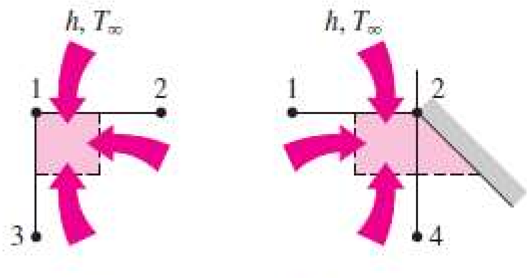
$$-k\frac{\Delta y}{2}\frac{(T_2-T_1)}{\Delta x} - k\Delta x\frac{(T_2-T_4)}{\Delta y} = h_i\frac{\Delta x}{2}(T_2 - T_i)$$

Taking $\Delta x = \Delta y$, it simplifies to

$$T_2\left(\frac{\Delta x h_i}{2k} + \frac{3}{2}\right) = \frac{\Delta x h_i}{2k}T_i + \left(T_4 + \frac{1}{2}T_1\right)$$

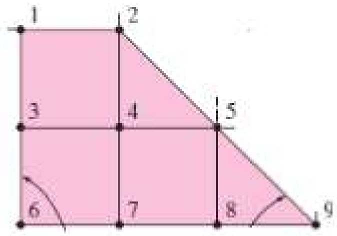
$$T_2(\frac{\Delta x h_i}{k} + 3) = \frac{\Delta x h_i}{k} T_i + (2T_4 + T_1)$$

$$T_2 = \frac{(\frac{\Delta x h_i}{k})T_i + (2T_4 + T_1)}{(\frac{\Delta x h_i}{k} + 3)}$$



(a) Node 1 (b) Node 2

Nodes 3,



$$-k\frac{\Delta x}{2}\frac{(T_3 - T_1)}{\Delta y} - k\Delta y\frac{(T_3 - T_4)}{\Delta x} - k\frac{\Delta x}{2}\frac{(T_3 - T_6)}{\Delta y} = 0$$

$$-2T_3 + \frac{1}{2}T_1 + T_4 + \frac{1}{2}T_6 = 0$$

$$T_1 + 2T_4 + T_6 - 4T_3 = 0$$

$$T_3 = \frac{T_1 + 2T_4 + T_6}{4}$$

Node 4,

$$-k\Delta x\frac{(T_4 - T_2)}{\Delta y} - k\Delta y\frac{(T_4 - T_3)}{\Delta x} - k\Delta y\frac{(T_4 - T_5)}{\Delta x} - k\Delta x\frac{(T_4 - T_7)}{\Delta y} = 0$$

$$T_2 + T_3 + T_5 + T_7 - 4T_4 = 0$$

$$T_4 = \frac{T_2 + T_3 + T_5 + T_7}{4}$$

Node 5,

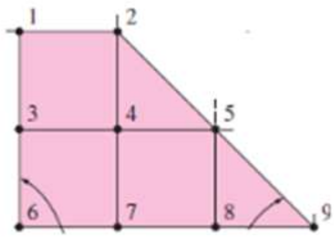
$$-k\Delta y\frac{(T_5 - T_4)}{\Delta x} - k\Delta x\frac{(T_5 - T_8)}{\Delta y} = 0$$

$$T_4 + T_8 - 2T_5 = 0$$

$$T_5 = \frac{T_4 + T_8}{2}$$

Node 6. (On the outer boundary, subjected to convection and radiation)

$$-k\frac{\Delta x}{2}\frac{(T_6 - T_3)}{\Delta y} - k\frac{\Delta y}{2}\frac{(T_6 - T_7)}{\Delta x} = h_o\frac{\Delta x}{2}(T_6 - T_o) + \varepsilon\sigma\frac{\Delta x}{2}\left(T_6{}^4 - T_{sky}{}^4\right)$$



Taking$\Delta x = \Delta y$, it simplifies to

$$T_6\left(2 + \frac{h_o \Delta x}{k}\right) = T_3 + T_7 + \frac{h_o \Delta x}{k} T_o + \varepsilon\sigma \frac{\Delta x}{k}\left(T_6{}^4 - T_{sky}{}^4\right)$$

$$T_6 = \frac{T_3 + T_7 + \frac{h_o \Delta x}{k} T_o + \varepsilon\sigma \frac{\Delta x}{k}\left(T_6{}^4 - T_{sky}{}^4\right)}{\left(2 + \frac{h_o \Delta x}{k}\right)}$$

Node 7. (On the outer boundary, subjected to convection and radiation, Fig

$$-k\Delta x \frac{(T_7 - T_4)}{\Delta y} - k\frac{\Delta y}{2}\frac{(T_7 - T_6)}{\Delta x} - k\frac{\Delta y}{2}\frac{(T_7 - T_8)}{\Delta x} = h_o \Delta x (T_7 - T_o) + \varepsilon\sigma\Delta x\left(T_7{}^4 - T_{sky}{}^4\right)$$

Taking$\Delta x = \Delta y$, it simplifies to

$$T_7\left(4 + \frac{2h_o \Delta x}{k}\right) = 2T_4 + T_6 + T_8 + \frac{2h_o \Delta x}{k} T_o + 2\varepsilon\sigma \frac{\Delta x}{k}\left(T_7{}^4 - T_{sky}{}^4\right)$$

$$T_7 = \frac{2T_4 + T_6 + T_8 + \frac{2h_o \Delta x}{k} T_o + 2\varepsilon\sigma \frac{\Delta x}{k}\left(T_7{}^4 - T_{sky}{}^4\right)}{\left(4 + \frac{2h_o \Delta x}{k}\right)}$$

Node 8.

$$T_8\left(4 + \frac{2h_o \Delta x}{k}\right) = 2T_5 + T_7 + T_9 + \frac{2h_o \Delta x}{k} T_o + 2\varepsilon\sigma \frac{\Delta x}{k}\left(T_8{}^4 - T_{sky}{}^4\right)$$

$$T_8 = \frac{2T_5 + T_7 + T_9 + \frac{2h_o \Delta x}{k} T_o + 2\varepsilon\sigma \frac{\Delta x}{k}\left(T_8{}^4 - T_{sky}{}^4\right)}{\left(4 + \frac{2h_o \Delta x}{k}\right)}$$

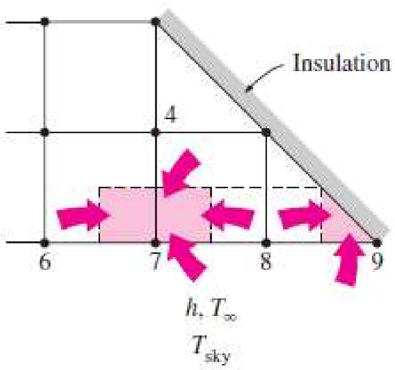
Node 9. (On the outer boundary, subjected to convection and radiation, Fig.

$$-k\frac{\Delta y}{2}\frac{(T_9 - T_8)}{\Delta x} = h_o \frac{\Delta x}{2}(T_9 - T_o) + \varepsilon\sigma \frac{\Delta x}{2}\left(T_9{}^4 - T_{sky}{}^4\right)$$

Taking$\Delta x = \Delta y$, it simplifies to

$$T_9\left(1 + \frac{h_o \Delta x}{k}\right) = T_8 + h_o \frac{\Delta x}{k} T_o + \varepsilon\sigma \frac{\Delta x}{k}\left(T_9{}^4 - T_{sky}{}^4\right)$$

$$T_9 = \frac{T_8 + h_o \frac{\Delta x}{k} T_o + \varepsilon\sigma \frac{\Delta x}{k}\left(T_9{}^4 - T_{sky}{}^4\right)}{\left(1 + \frac{h_o \Delta x}{k}\right)}$$



This problem involves radiation, which requires the use of absolute temperature, and thus all temperatures should be expressed in Kelvin. Alternately, we could use °C for all temperatures provided that the four temperatures in the radiation terms are expressed in the form $(T + 273)^4$. Substituting the given quantities, the system of nine equations for the determination of nine unknown nodal temperatures in a form suitable for use with the Gauss-Seidel iteration method becomes

$$T_1 = (T_2 + T_3 + 2865)/7$$
$$T_2 = (T_1 + 2T_4 + 2865)/8$$
$$T_3 = (T_1 + 2T_4 + T_6)/4$$
$$T_4 = (T_2 + T_3 + T_5 + T_7)/4$$
$$T_5 = (2T_4 + 2T_8)/4$$
$$T_6 = (T_2 + T_3 + 456.2 - 0.3645 \times 10^{-9}\, T_6^4)/3.5$$
$$T_7 = (2T_4 + T_6 + T_8 + 912.4 - 0.729 \times 10^{-9}\, T_7^4)/7$$
$$T_8 = (2T_5 + T_7 + T_9 + 912.4 - 0.729 \times 10^{-9}\, T_8^4)/7$$
$$T_9 = (T_8 + 456.2 - 0.3645 \times 10^{-9}\, T_9^4)/2.5$$

Solution Techniques
The Matrix Inversion Method

From the foregoing discussion we have seen that the numerical method is simply a means of approximating a continuous temperature distribution with the finite nodal elements. The more nodes taken, the closer the approximation; but, of course, more equations mean more cumbersome solutions. Fortunately, computers and even programmable calculators have the capability to obtain these solutions very quickly.
In practical problems the selection of a large number of nodes may be unnecessary because of uncertainties in boundary conditions. For example, it is not uncommon to have uncertainties in h, the convection coefficient, of ±15 to 20 percent. The nodal equations may be written as

$$\begin{aligned}
a_{11}T_1 + a_{12}T_2 + \cdots + a_{1n}T_n &= C_1 \\
a_{21}T_1 + a_{22}T_2 + \cdots \qquad &= C_2 \\
a_{31}T_1 + \cdots \qquad &= C_3 \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
a_{n1}T_1 + a_{n2}T_2 + \cdots + a_{nn}T_n &= C_n
\end{aligned} \qquad 4.25$$

where T_1, T_2, . . . , T_n are the unknown nodal temperatures. By using the matrix notation

$$[A] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & \\ a_{31} & & \cdots & \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \qquad [C] = \begin{bmatrix} C_1 \\ C_2 \\ \cdot \\ \cdot \\ \cdot \\ C_n \end{bmatrix} \qquad [T] = \begin{bmatrix} T_1 \\ T_2 \\ \cdot \\ \cdot \\ \cdot \\ T_n \end{bmatrix}$$

Equation (4.25) can be expressed as

$$[A][T] = [C] \qquad 4.26$$

and the problem is to find the inverse of $[A]$ such that

$$[T] = [A]^{-1}[C] \qquad 4.27$$

Designating $[A]^{-1}$ by

$$[A]^{-1} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & \\ \cdots & \cdots & \cdots & \cdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix}$$

the final solutions for the unknown temperatures are written in expanded form as

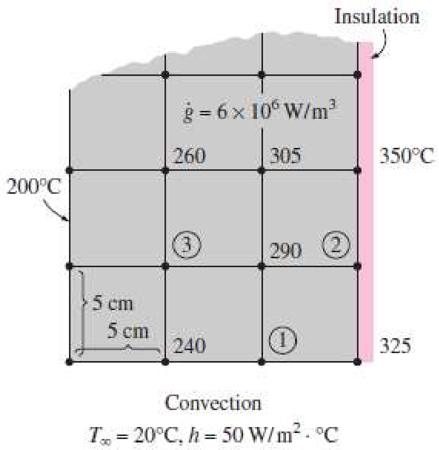
$$\begin{aligned} T_1 &= b_{11}C_1 + b_{12}C_2 + \cdots + b_{1n}C_n \\ T_2 &= b_{21}C_1 + \cdots \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ T_n &= b_{n1}C_1 + b_{n2}C_2 + \cdots + b_{nn}C_n \end{aligned} \qquad 4.28$$

Clearly, the larger the number of nodes, the more complex and time-consuming the solution, even with a high-speed computer. For most conduction problems the matrix contains a large number of zero elements so that some simplification in the procedure is afforded. For example, the matrix notation for the system of Example 4.1 would be

$$\begin{bmatrix} -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & -4.67 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 2 & -4.67 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 & 0 & -4.67 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 1 & -4.67 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -2.67 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \\ T_9 \end{bmatrix} = \begin{bmatrix} -600 \\ -500 \\ -567 \\ -100 \\ 0 \\ -67 \\ -167 \\ -67 \\ -67 \end{bmatrix}$$

Example 4.3:

Consider steady two-dimensional heat transfer in a long solid body whose cross section is given in the figure. The temperatures at the selected nodes and the thermal conditions at the boundaries are as shown. The thermal conductivity of the body is k = 214 W/m · °C, and heat is generated in the body uniformly at a rate of $\dot{q}$ =6*10^6 W/m^3. With a mesh size of Δx=Δy = 5.0 cm, determine(a) the temperatures at nodes 1, 2, and 3 and (b) the rate of heat loss from the bottom surface through a 1-m-long section of the body.



a:

Node 1:

$$-k\frac{\Delta y}{2}\frac{(T_1-325)}{\Delta x}-k\frac{\Delta y}{2}\frac{(T_1-240)}{\Delta x}-k\Delta x\frac{(T_1-290)}{\Delta y}+\dot{q}(\Delta x\Delta y)=h\Delta x(T_1-T_\infty)$$

$$T_1\left(4+2\frac{h\Delta x}{k}\right)=325+240+2(290)+\frac{2\dot{q}(\Delta x\Delta y)}{k}+\frac{2h\Delta x}{k}T_\infty$$

$$T_1=\frac{325+240+2(290)+\frac{2\dot{q}(\Delta x\Delta y)}{k}+\frac{2h\Delta x}{k}T_\infty}{\left(4+2\frac{h\Delta x}{k}\right)}$$

$T_1 = 319$ ˚C.

Node 2:

$$-k\frac{\Delta x}{2}\frac{(T_2-350)}{\Delta y}-k\frac{\Delta x}{2}\frac{(T_2-325)}{\Delta y}-k\Delta y\frac{(T_2-290)}{\Delta x}+\dot{q}(\Delta x\Delta y)=0$$

$$4T_2=350+325+2(290)+\frac{2\dot{q}(\Delta x\Delta y)}{k}$$

$$T_2=\frac{350+325+2(290)+\frac{2\dot{q}(\Delta x\Delta y)}{k}}{4}$$

$T_2 = 348.8$ ˚C.

Node 3:

$$290+200+260+240-4T_3+\frac{\dot{q}(\Delta x\Delta y)}{k}=0$$

$$T_3=\frac{290+200+260+240+\frac{\dot{q}(\Delta x\Delta y)}{k}}{4}$$

$T_3 = 265$ ˚C.

b:

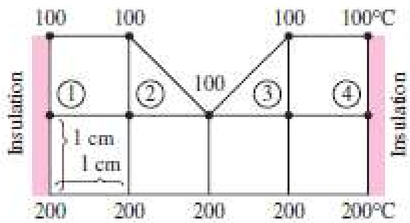
$q = h\frac{\Delta x}{2}(200 - T_\infty) + h\Delta x(240 - T_\infty) + h\Delta x(T_1 - T_\infty) + h\frac{\Delta x}{2}(325 - T_\infty)$

$q = 50\frac{0.05}{2}(200 - 20) + 50 * 0.05(240 - 20) + 50 * 0.05(319 - 20) + 50\frac{0.05}{2}(325 - 20)$

$q = 1903.75$ W

5–52 Consider steady two-dimensional heat transfer in a long solid bar whose cross section is given in the figure. The measured temperatures at selected points on the outer surfaces are as shown. The thermal conductivity of the body is k = 20 W/m · °C, and there is no heat generation. Using the finite difference method with a mesh size of $\Delta x = \Delta y$ =1.0 cm, determine the temperatures at the indicated points in the medium. Answers: $T_1 = T_4$ = 143°C, $T_2 = T_3$ = 136°C.

solution:



node 1:

$$-k\Delta y\frac{T_1 - T_2}{\Delta x} - k\frac{\Delta x}{2}\frac{T_1 - 100}{\Delta y} - k\frac{\Delta x}{2}\frac{T_1 - 200}{\Delta y} = 0$$

$$\frac{1}{2}(200 + 100) + T_2 - 2T_1 = 0$$

$$T_1 = \frac{150 + T_2}{2} \qquad (1)$$

Node 2:

$$-k\Delta y\frac{T_2 - T_1}{\Delta x} - k\Delta y\frac{T_2 - 100}{\Delta x} - k\Delta x\frac{T_2 - 100}{\Delta y} - k\Delta x\frac{T_2 - 200}{\Delta y} = 0$$

$$T_1 + 100 + 100 + 200 - 4T_2 = 0$$

$$T_2 = \frac{T_1 + 100 + 100 + 200}{4} \qquad (2)$$

Substitute equation (2) in equation (1):

$$T_1 = \frac{150 + \frac{T_1 + 100 + 100 + 200}{4}}{2}$$

$$T_1 = 75 + 50 + \frac{T_1}{8}$$

$$T_1 = \frac{125}{1 - \frac{1}{8}} = 143 \text{ ˚C}$$

$$T_1 = T_4 = 143 \text{ ˚C}$$

$$T_2 = T_3 = \frac{143 + 100 + 100 + 200}{4} = 136 \text{ ˚C}$$

5–61 Consider a long solid bar whose thermal conductivity is k = 12 W/m · °C and whose cross section is given in the figure. The top surface of the bar is maintained at 50°C while the bottom surface is maintained at 120°C. The left surface is insulated and the remaining three surfaces are subjected to convection with ambient air at T_∞ = 25°C with a heat transfer coefficient of h = 30 W/m^2 · °C. Using the finite difference method with a mesh size of $\Delta x = \Delta y$ = 10 cm, (*a*) obtain the finite difference formulation of this problem for steady two dimensional heat transfer and (*b*) determine the unknown nodal temperatures by solving those equations. *Answers:* (*b*) 85.7°C, 86.4°C, 87.6°C

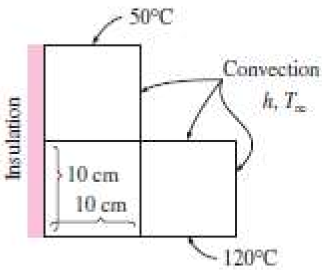
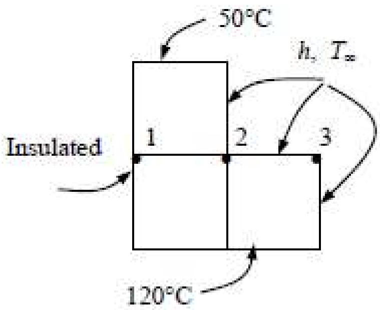


FIGURE P5–61

Solution:



Node 1:

$$-k\Delta y\frac{(T_1-T_2)}{\Delta x}-k\frac{\Delta x}{2}\frac{(T_1-50)}{\Delta y}-k\frac{\Delta x}{2}\frac{(T_1-120)}{\Delta y}=0$$

$$T_2+\frac{50}{2}+\frac{120}{2}-2T_1=0$$

$$-2T_1+T_2=-85 \qquad (1)$$

Node 2:

$$-k\frac{\Delta y}{2}\frac{(T_2-T_3)}{\Delta x}-k\Delta y\frac{(T_2-T_1)}{\Delta x}-k\frac{\Delta x}{2}\frac{(T_2-50)}{\Delta y}-k\Delta x\frac{(T_2-120)}{\Delta y}=h\frac{\Delta x}{2}(T_2-T_\infty)+h\frac{\Delta y}{2}(T_2-T_\infty)$$

$$T_1-\left(3+h\frac{\Delta x}{k}\right)T_2+\frac{1}{2}T_3=-145-h\frac{\Delta x}{k}T_\infty$$

$T_1 - \left(3 + 30\frac{0.1}{12}\right)T_2 + \frac{1}{2}T_3 = -145 - 30\frac{0.1}{12}(25)$

$T_1 - 3.25T_2 + 0.5T_3 = -151.25$ (2)

Node 3:

$-k\frac{\Delta y}{2}\frac{(T_3-T_2)}{\Delta x} - k\frac{\Delta x}{2}\frac{(T_3-120)}{\Delta y} = h\frac{\Delta x}{2}(T_3 - T_\infty) + h\frac{\Delta y}{2}(T_3 - T_\infty)$

$0.5T_2 - \left(1 + h\frac{\Delta x}{k}\right)T_3 = -60 - h\frac{\Delta x}{k}T_\infty$

$0.5T_2 - \left(1 + 30\frac{0.1}{12}\right)T_3 = -60 - 30\frac{0.1}{12}(25)$

$0.5T_2 - 1.25T_3 = -66.25$ (3)

Three equations with three anouns can be solved using matrix invers method.

$-2T_1 + T_2 = 85$

$T_1 - 3.25T_2 + 0.5T_3 = -151.25$

$0.5T_2 - 1.25T_3 = -66.25$

[A]= $\begin{bmatrix} -2 & 1 & 0 \\ 1 & -3.25 & 0.5 \\ 0 & 0.5 & -1.25 \end{bmatrix}$

[C]= $\begin{bmatrix} -85 \\ -151.25 \\ -66.25 \end{bmatrix}$

Det[A]= $\begin{bmatrix} -2 & 1 & 0 \\ 1 & -3.25 & 0.5 \\ 0 & 0.5 & -1.25 \end{bmatrix}\begin{matrix} -2 & 1 \\ 1 & -3.25 \\ 0 & 0.5 \end{matrix}$

=[(-2*-3.25*-1.25)+(1*0.5*0)+(0*1*0.5)]-[(0*-3.25*0)+(0.5*0.5*-2)+(-1.25*1*1)]

=[-8.125-(-0.5)-(-1.25)]=-6.375

Adj[A]= $\begin{bmatrix} \begin{bmatrix} -3.25 & 0.5 \\ 0.5 & -1.25 \end{bmatrix} & -\begin{bmatrix} 1 & 0 \\ 0.5 & -1.25 \end{bmatrix} & \begin{bmatrix} 1 & 0 \\ -3.25 & 0.5 \end{bmatrix} \\ -\begin{bmatrix} 1 & 0.5 \\ 0 & -1.25 \end{bmatrix} & \begin{bmatrix} -2 & 0 \\ 0 & -1.25 \end{bmatrix} & -\begin{bmatrix} -2 & 0 \\ 1 & 0.5 \end{bmatrix} \\ \begin{bmatrix} 1 & -3.25 \\ 0 & 0.5 \end{bmatrix} & -\begin{bmatrix} -2 & 1 \\ 0 & 0.5 \end{bmatrix} & \begin{bmatrix} -2 & 1 \\ 1 & -3.25 \end{bmatrix} \end{bmatrix}$

$\begin{bmatrix} -3.25 & 0.5 \\ 0.5 & -1.25 \end{bmatrix}$=[(-3.25*-1.25)-(0.5*0.5)]=3.8125

$\begin{bmatrix} 1 & 0.5 \\ 0 & -1.25 \end{bmatrix}$=[(1*-1.25)-(0.5*0)]=-1.25

$\begin{bmatrix} 1 & -3.25 \\ 0 & 0.5 \end{bmatrix}$ =[(1*0.5)-(0*-3.25)]=0.5

$\begin{bmatrix} 1 & 0 \\ 0.5 & -1.25 \end{bmatrix}$ =[(1*-1.25)-(0*0.5)]=-1.25

$\begin{bmatrix} -2 & 0 \\ 0 & -1.25 \end{bmatrix}$ =[(-2*-1.25)-(0)]=2.5

$\begin{bmatrix} -2 & 1 \\ 0 & 0.5 \end{bmatrix} = [(-2 * 0.5) - (0 * 1)] = -1$

$\begin{bmatrix} 1 & 0 \\ -3.25 & 0.5 \end{bmatrix}$ =[(1*0.5)-(0*-3.25)]=0.5

$\begin{bmatrix} -2 & 0 \\ 1 & 0.5 \end{bmatrix}$ =[(-2*0.5)-(0*1)]=-1

$\begin{bmatrix} -2 & 1 \\ 1 & -3.25 \end{bmatrix}$ =[(-2*-3.25)-(1*1)]=5.5

$$\text{Adj}[A]=\begin{bmatrix} 3.8125 & 1.25 & 0.5 \\ 1.25 & 2.5 & 1 \\ 0.5 & 1 & 5.5 \end{bmatrix}$$

$$[A]^{-1}=\frac{\text{Adj}[A]}{\text{Det}[A]}=\begin{bmatrix} -0.598 & -0.196 & -0.0784 \\ -0.196 & -0.392 & -0.157 \\ -0.0784 & -0.157 & -0.862 \end{bmatrix}$$

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}=\begin{bmatrix} -0.598 & -0.196 & -0.0784 \\ -0.196 & -0.392 & -0.157 \\ -0.0784 & -0.157 & -0.862 \end{bmatrix} * \begin{bmatrix} -85 \\ -151.25 \\ -66.25 \end{bmatrix}$$

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 85.7 \\ 86.35 \\ 87.5 \end{bmatrix}$$